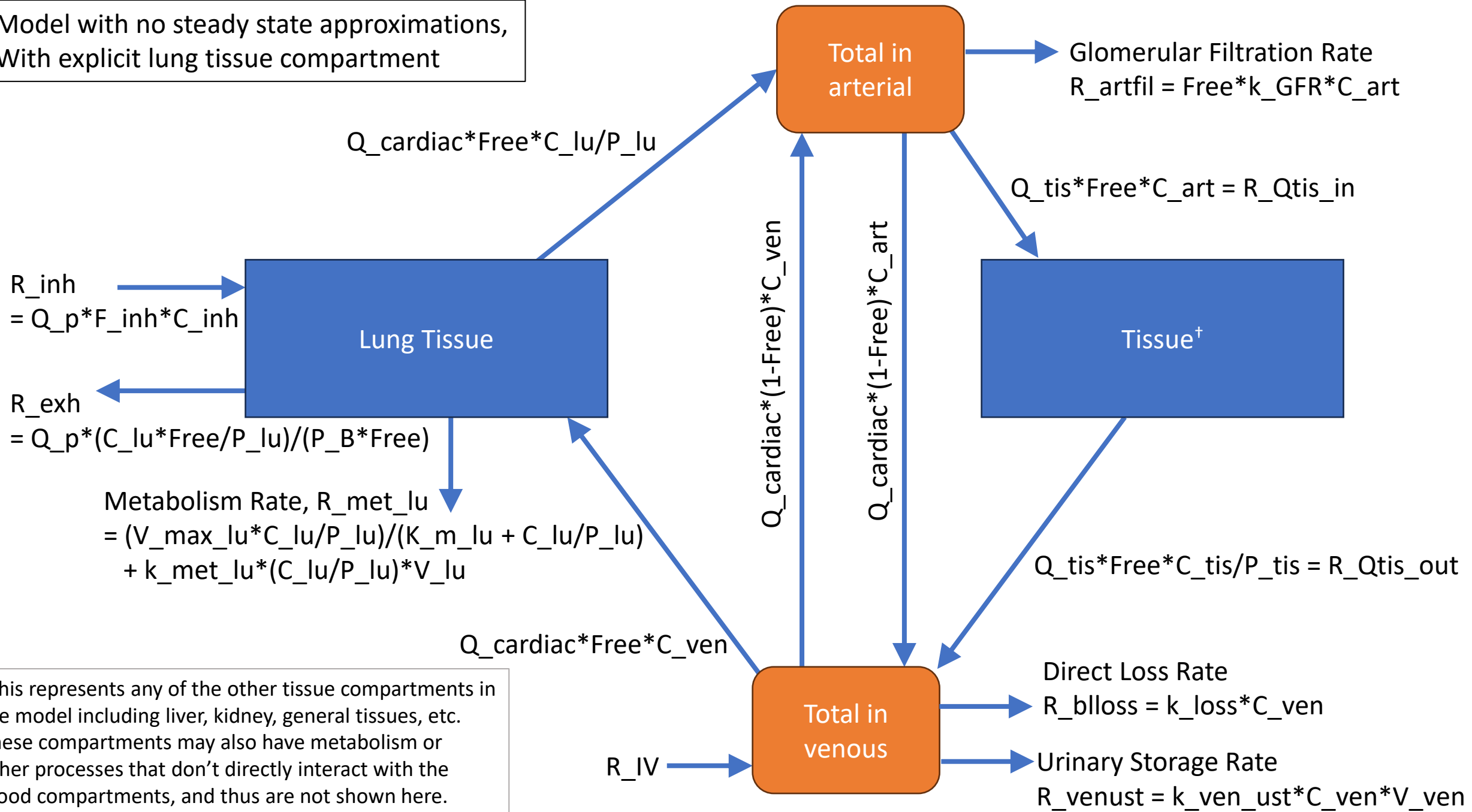


Model with no steady state approximations,
With explicit lung tissue compartment



Model with **no** steady state approximations

venous_ss = 0, arterial_ss = 0, GE_ss = 0, exist_lung = 1

$$R_{\text{loss}} = k_{\text{ven_ust}} * C_{\text{ven}} * V_{\text{ven}} + k_{\text{loss}} * C_{\text{ven}}$$

$$R_{\text{artfil}} = \text{Free} * k_{\text{GFR}} * C_{\text{art}}$$

$$dt(A_{\text{tis}}) = R_{\text{Qtis_in}} - R_{\text{Qtis_out}}$$

$$\begin{aligned} dt(A_{\text{ven}}) &= R_{\text{Qtis_out}} + R_{\text{IV}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} \\ &\quad - Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} - Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{ven}} - R_{\text{loss}} \\ &= R_{\text{Qtis_out}} + R_{\text{IV}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} - Q_{\text{cardiac}} * C_{\text{ven}} - R_{\text{loss}} \end{aligned}$$

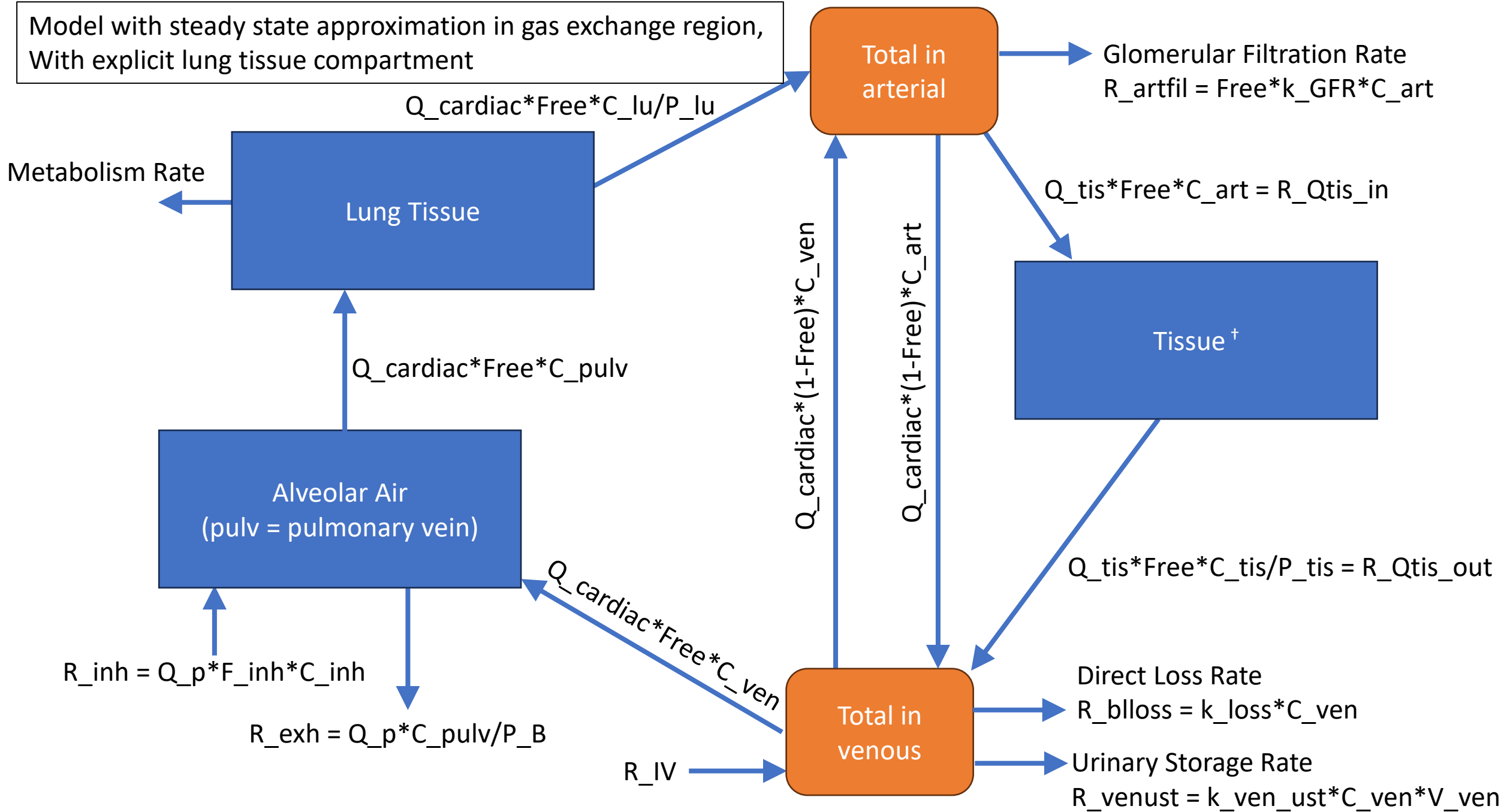
$$\begin{aligned} dt(A_{\text{lu}}) &= R_{\text{inh}} - R_{\text{exh}} + Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} - Q_{\text{cardiac}} * \text{Free} * C_{\text{lu}} / P_{\text{lu}} - R_{\text{met_lu}} \\ &= Q_{\text{p}} * F_{\text{inh}} * C_{\text{inh}} - Q_{\text{p}} * (C_{\text{lu}} / P_{\text{lu}}) / P_{\text{B}} + Q_{\text{cardiac}} * \text{Free} * (C_{\text{ven}} - C_{\text{lu}} / P_{\text{lu}}) - R_{\text{met_lu}} \end{aligned}$$

At steady state with no metabolism,

$$C_{\text{lu}} / P_{\text{lu}} = (Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} + Q_{\text{p}} * F_{\text{inh}} * C_{\text{inh}}) / (Q_{\text{cardiac}} * \text{Free} + Q_{\text{p}} / P_{\text{B}})$$

$$\begin{aligned} dt(A_{\text{art}}) &= Q_{\text{cardiac}} * \text{Free} * C_{\text{lu}} / P_{\text{lu}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{ven}} \\ &\quad - R_{\text{Qtis_in}} - Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} - R_{\text{artfil}} \end{aligned}$$

Model with steady state approximation in gas exchange region,
With explicit lung tissue compartment



Model with steady state approximation in gas exchange region,
With explicit lung tissue compartment
venous_ss = 0, arterial_ss = 0, GE_ss = 1, exist_lung = 1

$$\begin{aligned} R_{\text{loss}} &= k_{\text{ven_ust}} * C_{\text{ven}} * V_{\text{ven}} + k_{\text{loss}} * C_{\text{ven}} \\ R_{\text{artfil}} &= \text{Free} * k_{\text{GFR}} * C_{\text{art}} \end{aligned}$$

$$dt(A_{\text{tis}}) = R_{\text{Qtis_in}} - R_{\text{Qtis_out}}$$

$$\begin{aligned} dt(A_{\text{ven}}) &= R_{\text{Qtis_out}} + R_{\text{IV}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} \\ &\quad - Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} - Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{ven}} - R_{\text{loss}} \\ &= R_{\text{Qtis_out}} + R_{\text{IV}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} - Q_{\text{cardiac}} * C_{\text{ven}} - R_{\text{loss}} \end{aligned}$$

$$\begin{aligned} Dt(A_{\text{pulv}}) &= Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} + R_{\text{inh}} - Q_{\text{cardiac}} * \text{Free} * C_{\text{pulv}} - R_{\text{exh}} \\ &= Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} + Q_{\text{p}} * F_{\text{inh}} * C_{\text{inh}} - Q_{\text{cardiac}} * \text{Free} * C_{\text{pulv}} - Q_{\text{p}} * C_{\text{pulv}} / P_{\text{B}} \end{aligned}$$

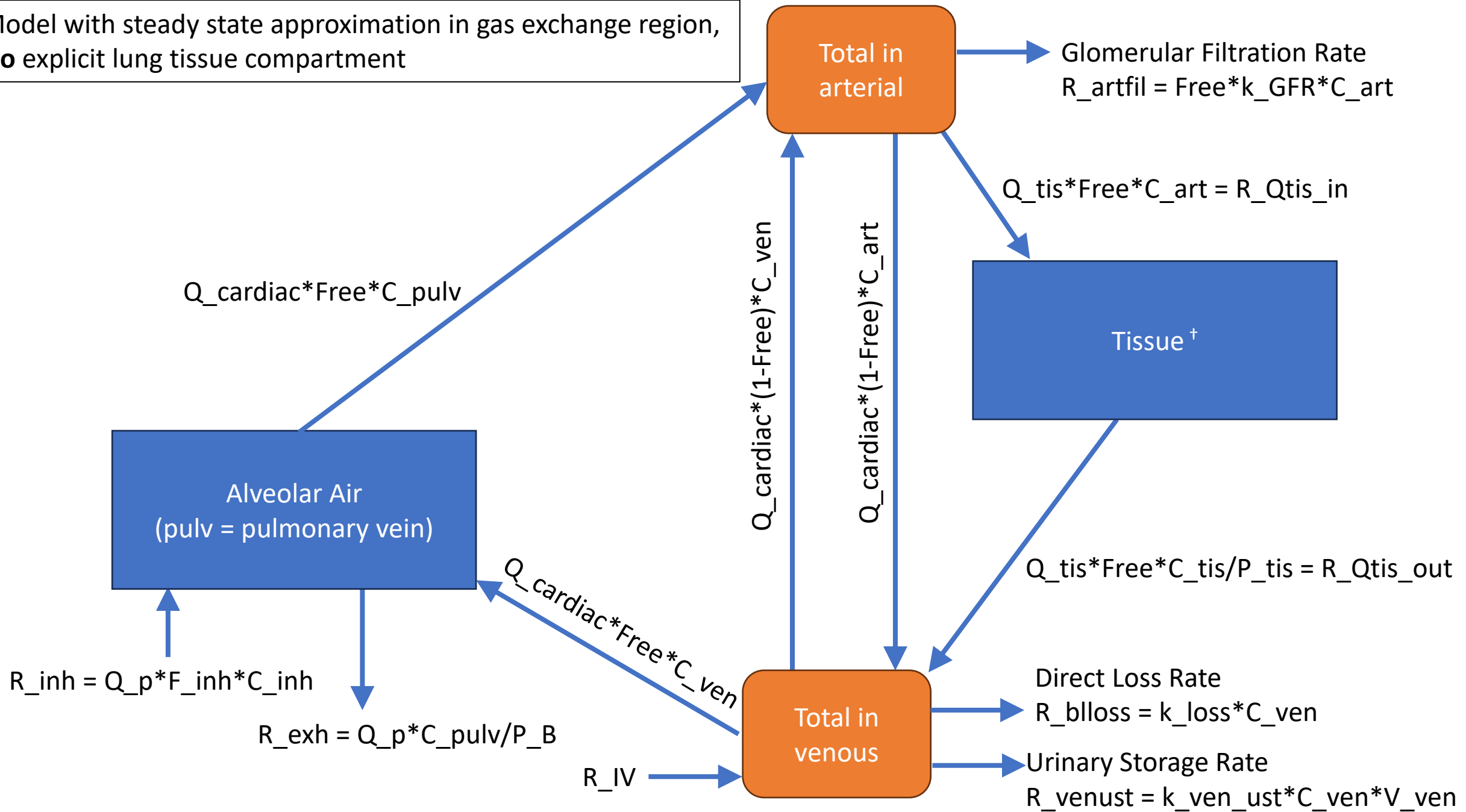
Set equal to zero and solve for C_pulv,

$$C_{\text{pulv}} = (Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} + Q_{\text{p}} * F_{\text{inh}} * C_{\text{inh}}) / (Q_{\text{cardiac}} * \text{Free} + Q_{\text{p}} / P_{\text{B}})$$

$$dt(A_{\text{lu}}) = Q_{\text{cardiac}} * \text{Free} * C_{\text{pulv}} - Q_{\text{cardiac}} * \text{Free} * C_{\text{lu}} / P_{\text{lu}} - R_{\text{met_lu}}$$

$$\begin{aligned} dt(A_{\text{art}}) &= Q_{\text{cardiac}} * \text{Free} * C_{\text{lu}} / P_{\text{lu}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{ven}} \\ &\quad - R_{\text{Qtis_in}} - Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} - R_{\text{artfil}} \end{aligned}$$

Model with steady state approximation in gas exchange region,
No explicit lung tissue compartment



Model with steady state approximation in gas exchange region,
No explicit lung tissue compartment
venous_ss = 0, arterial_ss = 0, GE_ss = 1, exist_lung = 0

$$\begin{aligned} R_{\text{loss}} &= k_{\text{ven_ust}} * C_{\text{ven}} * V_{\text{ven}} + k_{\text{loss}} * C_{\text{ven}} \\ R_{\text{artfil}} &= \text{Free} * k_{\text{GFR}} * C_{\text{art}} \end{aligned}$$

$$dt(A_{\text{tis}}) = R_{\text{Qtis_in}} - R_{\text{Qtis_out}}$$

$$\begin{aligned} dt(A_{\text{ven}}) &= R_{\text{Qtis_out}} + R_{\text{IV}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} \\ &\quad - Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} - Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{ven}} - R_{\text{loss}} \\ &= R_{\text{Qtis_out}} + R_{\text{IV}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} - Q_{\text{cardiac}} * C_{\text{ven}} - R_{\text{loss}} \end{aligned}$$

$$Dt(A_{\text{pulv}}) = Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} + R_{\text{inh}} - Q_{\text{cardiac}} * \text{Free} * C_{\text{pulv}} - R_{\text{exh}}$$

****set equal to zero and solve for C_pulv****

$$C_{\text{pulv}} = (Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} + Q_{\text{p}} * F_{\text{inh}} * C_{\text{inh}}) / (Q_{\text{cardiac}} * \text{Free} + Q_{\text{p}} / P_{\text{B}})$$

$$\begin{aligned} dt(A_{\text{art}}) &= Q_{\text{cardiac}} * \text{Free} * C_{\text{pulv}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{ven}} - R_{\text{Qtis_in}} - Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} - R_{\text{artfil}} \\ &= Q_{\text{cardiac}} * \text{Free} * C_{\text{lu}} / P_{\text{lu}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{ven}} - Q_{\text{cardiac}} * C_{\text{art}} - R_{\text{artfil}} \end{aligned}$$

Implementing the steady state approximations for blood compartments (venous_ss = 1, arterial_ss = 1)...

$$\begin{aligned} dt(A_{\text{ven}}) &= R_{\text{Qtis_out}} + R_{\text{IV}} + Q_{\text{cardiac}}*(1-\text{Free})*C_{\text{art}} - Q_{\text{cardiac}}*C_{\text{ven}} - R_{\text{loss}} \\ &= R_{\text{Qtis_out}} + R_{\text{IV}} + Q_{\text{cardiac}}*(1-\text{Free})*C_{\text{art}} \\ &\quad - Q_{\text{cardiac}}*C_{\text{ven}} - k_{\text{ven_ust}}*V_{\text{ven}}*C_{\text{ven}} - k_{\text{loss}}*C_{\text{ven}} \end{aligned}$$

Steady state approx.:

$$C_{\text{ven}} = (R_{\text{Qtis_out}} + Q_{\text{cardiac}}*(1-\text{Free})*C_{\text{art}} + R_{\text{IV}})/(Q_{\text{cardiac}} + k_{\text{ven_ust}}*V_{\text{ven}} + k_{\text{loss}})$$

$$C_{\text{pulv}} = (Q_{\text{cardiac}}*\text{Free}*C_{\text{ven}} + Q_{\text{p}}*F_{\text{inh}}*C_{\text{inh}}) / (Q_{\text{cardiac}}*\text{Free} + Q_{\text{p}}/P_{\text{B}})$$

With explicit lung tissue:

$$\begin{aligned} dt(A_{\text{art}}) &= Q_{\text{cardiac}}*\text{Free}*C_{\text{lu}}/P_{\text{lu}} + Q_{\text{cardiac}}*(1-\text{Free})*C_{\text{ven}} - Q_{\text{cardiac}}*C_{\text{art}} - R_{\text{artfil}} \\ &= Q_{\text{cardiac}}*\text{Free}*C_{\text{lu}}/P_{\text{lu}} + Q_{\text{cardiac}}*(1-\text{Free})*C_{\text{ven}} - Q_{\text{cardiac}}*C_{\text{art}} - \text{Free}*k_{\text{GFR}}*C_{\text{art}} \end{aligned}$$

Steady state approx.:

$$C_{\text{art}} = (Q_{\text{cardiac}}*\text{Free}*C_{\text{lu}}/P_{\text{lu}} + Q_{\text{cardiac}}*(1-\text{Free})*C_{\text{ven}})/(Q_{\text{cardiac}} + \text{Free}*k_{\text{GFR}})$$

With no explicit lung tissue:

$$\begin{aligned} dt(A_{\text{art}}) &= Q_{\text{cardiac}}*\text{Free}*C_{\text{pulv}} + Q_{\text{cardiac}}*(1-\text{Free})*C_{\text{ven}} - Q_{\text{cardiac}}*C_{\text{art}} - R_{\text{artfil}} \\ &= Q_{\text{cardiac}}*\text{Free}*C_{\text{pulv}} + Q_{\text{cardiac}}*(1-\text{Free})*C_{\text{ven}} - Q_{\text{cardiac}}*C_{\text{art}} - \text{Free}*k_{\text{GFR}}*C_{\text{art}} \end{aligned}$$

Steady state approx.:

$$C_{\text{art}} = (Q_{\text{cardiac}}*\text{Free}*C_{\text{pulv}} + Q_{\text{cardiac}}*(1-\text{Free})*C_{\text{ven}})/(Q_{\text{cardiac}} + \text{Free}*k_{\text{GFR}})$$

Implementing the steady state approximations for blood compartments (venous_ss = 1, arterial_ss = 1)...

With explicit lung tissue and no approx. in gas exchange region (GE_ss = 0, exist_lung = 1):

$$C_{\text{ven}} = (R_{\text{Qtis_out}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} + R_{\text{IV}}) / (Q_{\text{cardiac}} + k_{\text{ven_ust}} * V_{\text{ven}} + k_{\text{loss}})$$

$$C_{\text{pulv}} = 0$$

$$C_{\text{art}} = (Q_{\text{cardiac}} * \text{Free} * C_{\text{lu}} / P_{\text{lu}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{ven}}) / (Q_{\text{cardiac}} + \text{Free} * k_{\text{GFR}})$$

Matrix form:

$$\begin{bmatrix} Q_{\text{cardiac}} + k_{\text{ven_ust}} * V_{\text{ven}} + k_{\text{loss}} & -Q_{\text{cardiac}}(1 - \text{Free}) \\ -Q_{\text{cardiac}}(1 - \text{Free}) & Q_{\text{cardiac}} + \text{Free} * k_{\text{GFR}} \end{bmatrix} \begin{bmatrix} C_{\text{ven}} \\ C_{\text{art}} \end{bmatrix} = \begin{bmatrix} R_{\text{IV}} + R_{\text{Qtis_out}} \\ Q_{\text{cardiac}} * \text{Free} * C_{\text{lu}} / P_{\text{lu}} \end{bmatrix}$$

With explicit lung tissue and steady state approx. in gas exchange region (GE_ss = 1, exist_lung = 1):

$$C_{\text{ven}} = (R_{\text{Qtis_out}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{art}} + R_{\text{IV}}) / (Q_{\text{cardiac}} + k_{\text{ven_ust}} * V_{\text{ven}} + k_{\text{loss}})$$

$$C_{\text{pulv}} = (Q_{\text{cardiac}} * \text{Free} * C_{\text{ven}} + Q_{\text{p}} * F_{\text{inh}} * C_{\text{inh}}) / (Q_{\text{cardiac}} * \text{Free} + Q_{\text{p}} / P_{\text{B}})$$

$$C_{\text{art}} = (Q_{\text{cardiac}} * \text{Free} * C_{\text{lu}} / P_{\text{lu}} + Q_{\text{cardiac}} * (1 - \text{Free}) * C_{\text{ven}}) / (Q_{\text{cardiac}} + \text{Free} * k_{\text{GFR}})$$

Matrix form:

$$\begin{bmatrix} Q_{\text{cardiac}} + k_{\text{ven_ust}} * V_{\text{ven}} + k_{\text{loss}} & 0 & -Q_{\text{cardiac}}(1 - \text{Free}) \\ -Q_{\text{cardiac}} * \text{Free} & Q_{\text{cardiac}} * \text{Free} + Q_{\text{p}} / P_{\text{B}} & 0 \\ -Q_{\text{cardiac}}(1 - \text{Free}) & 0 & Q_{\text{cardiac}} + \text{Free} * k_{\text{GFR}} \end{bmatrix} \begin{bmatrix} C_{\text{ven}} \\ C_{\text{pulv}} \\ C_{\text{art}} \end{bmatrix} = \begin{bmatrix} R_{\text{IV}} + R_{\text{Qtis_out}} \\ Q_{\text{p}} * F_{\text{inh}} * C_{\text{inh}} \\ Q_{\text{cardiac}} * \text{Free} * C_{\text{lu}} / P_{\text{lu}} \end{bmatrix}$$

Implementing the steady state approximations for blood compartments (venous_ss = 1, arterial_ss = 1)...

Without explicit lung tissue and with steady state approx. in gas exchange region (GE_ss = 1, exist_lung = 0):

$$C_{\text{ven}} = (R_{\text{Qtis_out}} + Q_{\text{cardiac}} \cdot (1 - \text{Free}) \cdot C_{\text{art}} + R_{\text{IV}}) / (Q_{\text{cardiac}} + k_{\text{ven_ust}} \cdot V_{\text{ven}} + k_{\text{loss}})$$

$$C_{\text{pulv}} = (Q_{\text{cardiac}} \cdot \text{Free} \cdot C_{\text{ven}} + Q_{\text{p}} \cdot F_{\text{inh}} \cdot C_{\text{inh}}) / (Q_{\text{cardiac}} \cdot \text{Free} + Q_{\text{p}} / P_{\text{B}})$$

$$C_{\text{art}} = (Q_{\text{cardiac}} \cdot \text{Free} \cdot C_{\text{pulv}} + Q_{\text{cardiac}} \cdot (1 - \text{Free}) \cdot C_{\text{ven}}) / (Q_{\text{cardiac}} + \text{Free} \cdot k_{\text{GFR}})$$

Matrix form:

$$\begin{bmatrix} Q_{\text{cardiac}} + k_{\text{ven_ust}} \cdot V_{\text{ven}} + k_{\text{loss}} & 0 & -Q_{\text{cardiac}}(1 - \text{Free}) \\ -Q_{\text{cardiac}} \cdot \text{Free} & Q_{\text{cardiac}} \cdot \text{Free} + Q_{\text{p}} / P_{\text{B}} & 0 \\ -Q_{\text{cardiac}}(1 - \text{Free}) & -Q_{\text{cardiac}} \cdot \text{Free} & Q_{\text{cardiac}} + \text{Free} \cdot k_{\text{GFR}} \end{bmatrix} \begin{bmatrix} C_{\text{ven}} \\ C_{\text{pulv}} \\ C_{\text{art}} \end{bmatrix} = \begin{bmatrix} R_{\text{IV}} + R_{\text{Qtis_out}} \\ Q_{\text{p}} \cdot F_{\text{inh}} \cdot C_{\text{inh}} \\ 0 \end{bmatrix}$$

To solve the matrix equations describing each case, we used YACAS (a symbolic algebra tool) online:
https://www.yacas.org/yacas_online/yacas_online.html

Relevant Documentation:

SolveMatrix function used to solve the matrix equation

https://yacas.readthedocs.io/en/latest/reference_manual/linear-algebra.html#SolveMatrix

CForm function used to convert to solution to C code

https://yacas.readthedocs.io/en/latest/reference_manual/io.html#CForm

Parameter names were simplified for use with YACAS as follows:

Name in Template Code	Name Used as Input to YACAS
Q_cardiac	Qc
Free	F
k_GFR	kG
C_lu	Clu
P_lu	Plu

Name in Template Code	Name Used as Input to YACAS
Q_p	Qp
P_B	PB
R_IV + R_Qtis_out	R
k_ven_ust*V_ven + k_loss	L
Q_p*F_inh*C_inh	Qin

Implementing the steady state approximations for blood compartments,
with explicit lung tissue and no approx. in the gas exchange region

Using YACAS (symbolic algebra tool) online: https://www.yacas.org/yacas_online/yacas_online.html

Relevant Documentation:

https://yacas.readthedocs.io/en/latest/reference_manual/linear-algebra.html#SolveMatrix

https://yacas.readthedocs.io/en/latest/reference_manual/io.html#CForm

$M := \{\{Q_c + L, -Q_c \cdot (1 - F)\}, \{-Q_c \cdot (1 - F), Q_c + F \cdot k_G\}\}$

$x := \{\{C_v\}, \{C_a\}\}$

$b := \{\{R\}, \{Q_c \cdot F \cdot C_{lu} / P_{lu}\}\}$

$x := \text{SolveMatrix}(M, b)$

$$\begin{aligned} R &= R_{IV} + R_{Qtis_out} \\ L &= k_{ven_ust} \cdot V_{ven} + k_{loss} \end{aligned}$$

$C_{Form}(x[1])$

$$C_{ven} = ((Q_c + F \cdot k_G) \cdot R + (Q_c \cdot (1 - F) \cdot Q_c \cdot F \cdot C_{lu}) / P_{lu}) / ((Q_c + L) \cdot (Q_c + F \cdot k_G) - \text{pow}(Q_c \cdot (1 - F), 2))$$

$C_{Form}(x[2])$

$$C_{art} = (((Q_c + L) \cdot Q_c \cdot F \cdot C_{lu}) / P_{lu} + Q_c \cdot (1 - F) \cdot R) / ((Q_c + L) \cdot (Q_c + F \cdot k_G) - \text{pow}(Q_c \cdot (1 - F), 2))$$

If $F=1$, $k_G=0$, $L=0$:

$$C_{ven} = ((Q_c + 1 \cdot 0) \cdot R + (Q_c \cdot (1 - 1) \cdot Q_c \cdot 1 \cdot C_{lu}) / P_{lu}) / ((Q_c + 0) \cdot (Q_c + 1 \cdot 0) - \text{pow}(Q_c \cdot (1 - 1), 2))$$

$$= ((Q_c) \cdot R) / ((Q_c) \cdot (Q_c)) = R / Q_c$$

$$C_{art} = (((Q_c + 0) \cdot Q_c \cdot 1 \cdot C_{lu}) / P_{lu} + Q_c \cdot (1 - 1) \cdot R) / ((Q_c + 0) \cdot (Q_c + 1 \cdot 0) - \text{pow}(Q_c \cdot (1 - 1), 2))$$

$$= (((Q_c) \cdot Q_c \cdot C_{lu}) / P_{lu}) / ((Q_c) \cdot (Q_c)) = C_{lu} / P_{lu}$$

Implementing the steady state approximations for blood compartments,

with explicit lung tissue and steady state approx. in the gas exchange region

Using YACAS (symbolic algebra tool) online: https://www.yacas.org/yacas_online/yacas_online.html

Relevant Documentation:

https://yacas.readthedocs.io/en/latest/reference_manual/linear-algebra.html#SolveMatrix

https://yacas.readthedocs.io/en/latest/reference_manual/io.html#CForm

$M := \{\{Q_c + L, 0, -Q_c \cdot (1 - F)\}, \{-Q_c \cdot F, Q_c \cdot F + Q_p / P_B, 0\}, \{-Q_c \cdot (1 - F), 0, Q_c + F \cdot k_G\}\}$

$x := \{\{C_v\}, \{C_p\}, \{C_a\}\}$

$b := \{\{R\}, \{Q_{in}\}, \{Q_c \cdot F \cdot C_{lu} / P_{lu}\}\}$

$x := \text{SolveMatrix}(M, b)$

$R = R_{IV} + R_{Qtis_out}$

$Q_{in} = Q_p \cdot F_{inh} \cdot C_{inh}$

$L = k_{ven_ust} \cdot V_{ven} + k_{loss}$

$CForm(x[1])$

$$\frac{(Q_c + F \cdot k_G) \cdot (Q_c \cdot F + Q_p / P_B) \cdot R + (Q_c \cdot (1 - F) \cdot (Q_c \cdot F + Q_p / P_B) \cdot Q_c \cdot F \cdot C_{lu}) / P_{lu}}{(Q_c + L) \cdot (Q_c \cdot F + Q_p / P_B) \cdot (Q_c + F \cdot k_G) - (Q_c \cdot F + Q_p / P_B) \cdot \text{pow}(Q_c \cdot (1 - F), 2)}$$

$CForm(x[2])$

$$\frac{(Q_c + F \cdot k_G) \cdot (Q_c + L) \cdot Q_{in} + (Q_c \cdot (1 - F) \cdot Q_c \cdot F \cdot Q_c \cdot F \cdot C_{lu}) / P_{lu} + (Q_c + F \cdot k_G) \cdot Q_c \cdot F \cdot R - Q_{in} \cdot \text{pow}(Q_c \cdot (1 - F), 2)}{(Q_c + L) \cdot (Q_c \cdot F + Q_p / P_B) \cdot (Q_c + F \cdot k_G) - (Q_c \cdot F + Q_p / P_B) \cdot \text{pow}(Q_c \cdot (1 - F), 2)}$$

$CForm(x[3])$

$$\frac{((Q_c + L) \cdot (Q_c \cdot F + Q_p / P_B) \cdot Q_c \cdot F \cdot C_{lu}) / P_{lu} + Q_c \cdot (1 - F) \cdot (Q_c \cdot F + Q_p / P_B) \cdot R}{(Q_c + L) \cdot (Q_c \cdot F + Q_p / P_B) \cdot (Q_c + F \cdot k_G) - (Q_c \cdot F + Q_p / P_B) \cdot \text{pow}(Q_c \cdot (1 - F), 2)}$$

Implementing the steady state approximations for blood compartments,
with explicit lung tissue and steady state approx. in the gas exchange region

$$\begin{aligned} R &= R_{IV} + R_{Qtis_out} \\ Q_{in} &= Q_p * F_{inh} * C_{inh} \\ L &= k_{ven_ust} * V_{ven} + k_{loss} \end{aligned}$$

From previous:

$$\begin{aligned} C_{ven} &= \left((Q_c + F * k_G) * (Q_c * F + Q_p / P_B) * R + (Q_c * (1 - F) * (Q_c * F + Q_p / P_B) * Q_c * F * C_{lu}) / P_{lu} \right) / \\ &\quad \left((Q_c + L) * (Q_c * F + Q_p / P_B) * (Q_c + F * k_G) - (Q_c * F + Q_p / P_B) * \text{pow}(Q_c * (1 - F), 2) \right) \\ C_{pulv} &= \left((Q_c + F * k_G) * (Q_c + L) * Q_{in} + (Q_c * (1 - F) * Q_c * F * Q_c * F * C_{lu}) / P_{lu} + (Q_c + F * k_G) * Q_c * F * R - Q_{in} * \text{pow}(Q_c * (1 - F), 2) \right) / \\ &\quad \left((Q_c + L) * (Q_c * F + Q_p / P_B) * (Q_c + F * k_G) - (Q_c * F + Q_p / P_B) * \text{pow}(Q_c * (1 - F), 2) \right) \\ C_{art} &= \left(((Q_c + L) * (Q_c * F + Q_p / P_B) * Q_c * F * C_{lu}) / P_{lu} + Q_c * (1 - F) * (Q_c * F + Q_p / P_B) * R \right) / \\ &\quad \left((Q_c + L) * (Q_c * F + Q_p / P_B) * (Q_c + F * k_G) - (Q_c * F + Q_p / P_B) * \text{pow}(Q_c * (1 - F), 2) \right) \end{aligned}$$

If $F=1$, $k_G=0$, $L=0$:

$$\begin{aligned} C_{ven} &= \left((Q_c + 1 * 0) * (Q_c * 1 + Q_p / P_B) * R + (Q_c * (1 - 1) * (Q_c * 1 + Q_p / P_B) * Q_c * 1 * C_{lu}) / P_{lu} \right) / \\ &\quad \left((Q_c + 0) * (Q_c * 1 + Q_p / P_B) * (Q_c + 1 * 0) - (Q_c * 1 + Q_p / P_B) * \text{pow}(Q_c * (1 - 1), 2) \right) \\ &= (Q_c * (Q_c + Q_p / P_B) * R) / ((Q_c) * (Q_c + Q_p / P_B) * (Q_c)) = R / Q_c \\ C_{pulv} &= \left((Q_c + 1 * 0) * (Q_c + 0) * Q_{in} + (Q_c * (1 - 1) * Q_c * 1 * Q_c * 1 * C_{lu}) / P_{lu} + (Q_c + 1 * 0) * Q_c * 1 * R - Q_{in} * \text{pow}(Q_c * (1 - 1), 2) \right) / \\ &\quad \left((Q_c + 0) * (Q_c * 1 + Q_p / P_B) * (Q_c + 1 * 0) - (Q_c * 1 + Q_p / P_B) * \text{pow}(Q_c * (1 - 1), 2) \right) \\ &= ((Q_c) * (Q_c) * Q_{in} + (Q_c) * Q_c * R) / ((Q_c) * (Q_c + Q_p / P_B) * (Q_c)) = (Q_{in} + R) / (Q_c + Q_p / P_B) \\ &= (Q_{in} + Q_c * C_{ven}) / (Q_c + Q_p / P_B) \\ C_{art} &= \left(((Q_c + 0) * (Q_c * 1 + Q_p / P_B) * Q_c * 1 * C_{lu}) / P_{lu} + Q_c * (1 - 1) * (Q_c * 1 + Q_p / P_B) * R \right) / \\ &\quad \left((Q_c + 0) * (Q_c * 1 + Q_p / P_B) * (Q_c + 1 * 0) - (Q_c * 1 + Q_p / P_B) * \text{pow}(Q_c * (1 - 1), 2) \right) \\ &= ((Q_c) * (Q_c + Q_p / P_B) * Q_c * C_{lu}) / P_{lu} / ((Q_c) * (Q_c + Q_p / P_B) * (Q_c)) = C_{lu} / P_{lu} \end{aligned}$$

Implementing the steady state approximations for blood compartments,
without explicit lung tissue

Using YACAS (symbolic algebra tool) online: https://www.yacas.org/yacas_online/yacas_online.html

Relevant Documentation:

https://yacas.readthedocs.io/en/latest/reference_manual/linear-algebra.html#SolveMatrix

https://yacas.readthedocs.io/en/latest/reference_manual/io.html#CForm

```
M:={{Qc+L,0,-Qc*(1-F)},{-Qc*F,Qc*F+Qp/PB,0},{-Qc*(1-F),-Qc*F,Qc+F*kG}}
```

```
x:={{Cv},{Cp},{Ca}}
```

```
b:={{R},{Qin},{0}}
```

```
x := SolveMatrix(M, b)
```

$$\begin{aligned} R &= R_{IV} + R_{Qtis_out} \\ Q_{in} &= Q_p * F_{inh} * C_{inh} \\ L &= k_{ven_ust} * V_{ven} + k_{loss} \end{aligned}$$

```
CForm(x[1])
```

```
( (Qc+F*kG)*(Qc*F+Qp/PB)*R+Qc*F*Qc*(1-F)*Qin ) /
```

```
( (Qc+L)*(Qc*F+Qp/PB)*(Qc+F*kG)-Qc*(1-F)*pow(Qc*F,2)-(Qc*F+Qp/PB)*pow(Qc*(1-F),2) )
```

```
CForm(x[2])
```

```
( (Qc+F*kG)*(Qc+L)*Qin+(Qc+F*kG)*Qc*F*R-Qin*pow(Qc*(1-F),2) ) /
```

```
( (Qc+L)*(Qc*F+Qp/PB)*(Qc+F*kG)-Qc*(1-F)*pow(Qc*F,2)-(Qc*F+Qp/PB)*pow(Qc*(1-F),2) )
```

```
CForm(x[3])
```

```
( Qc*F*(Qc+L)*Qin+R*pow(Qc*F,2)+Qc*(1-F)*(Qc*F+Qp/PB)*R ) /
```

```
( (Qc+L)*(Qc*F+Qp/PB)*(Qc+F*kG)-Qc*(1-F)*pow(Qc*F,2)-(Qc*F+Qp/PB)*pow(Qc*(1-F),2) )
```

Implementing the steady state approximations for blood compartments,
without explicit lung tissue

$$\begin{aligned} R &= R_{IV} + R_{Qtis_out} \\ Q_{in} &= Q_p * F_{inh} * C_{inh} \\ L &= k_{ven_ust} * V_{ven} + k_{loss} \end{aligned}$$

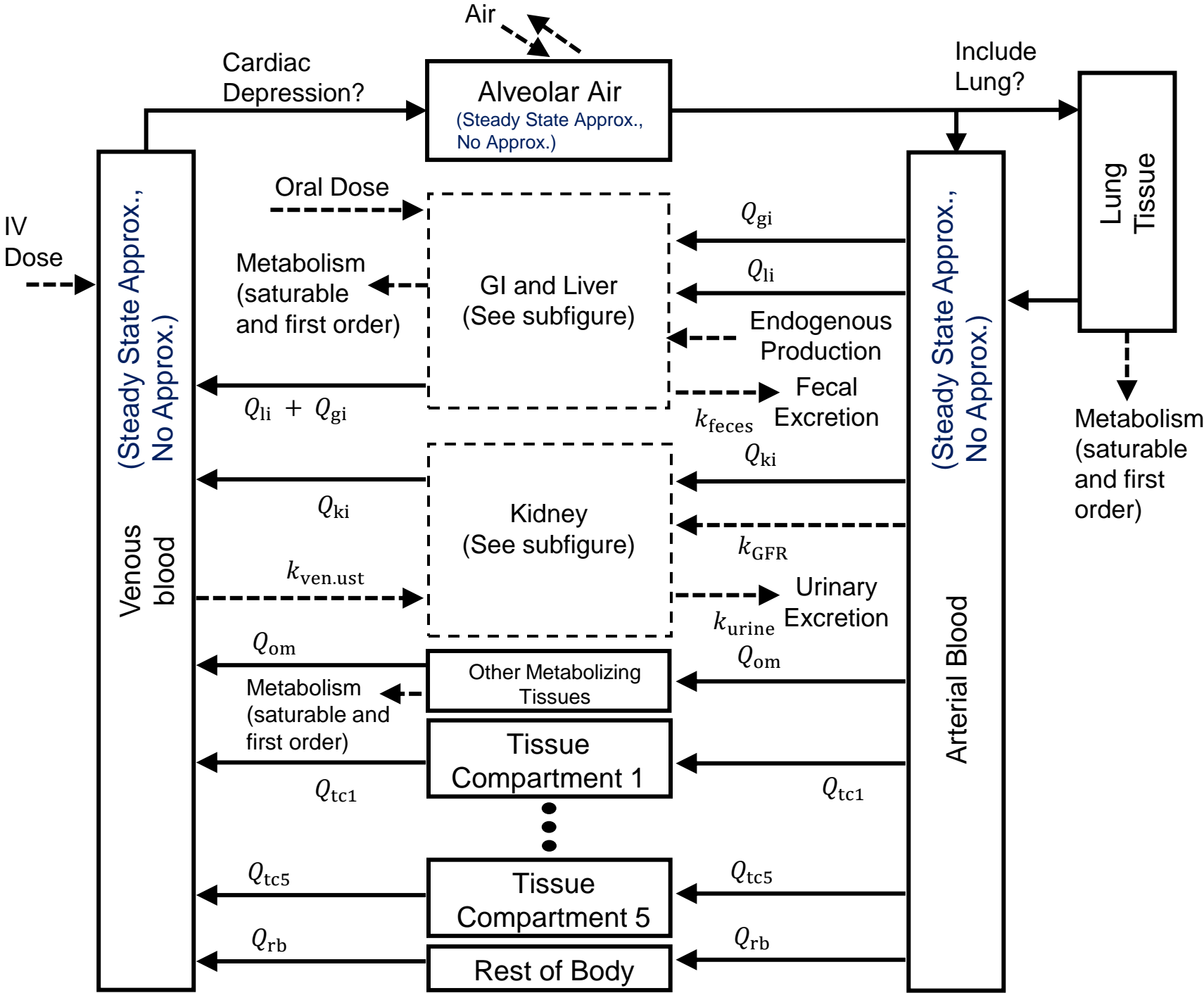
From previous:

$$\begin{aligned} C_{ven} &= ((Q_c + F * k_G) * (Q_c * F + Q_p / P_B) * R + Q_c * F * Q_c * (1 - F) * Q_{in}) / \\ &\quad ((Q_c + L) * (Q_c * F + Q_p / P_B) * (Q_c + F * k_G) - Q_c * (1 - F) * \text{pow}(Q_c * F, 2) - (Q_c * F + Q_p / P_B) * \text{pow}(Q_c * (1 - F), 2)) \\ C_{pulv} &= ((Q_c + F * k_G) * (Q_c + L) * Q_{in} + (Q_c + F * k_G) * Q_c * F * R - Q_{in} * \text{pow}(Q_c * (1 - F), 2)) / \\ &\quad ((Q_c + L) * (Q_c * F + Q_p / P_B) * (Q_c + F * k_G) - Q_c * (1 - F) * \text{pow}(Q_c * F, 2) - (Q_c * F + Q_p / P_B) * \text{pow}(Q_c * (1 - F), 2)) \\ C_{art} &= (Q_c * F * (Q_c + L) * Q_{in} + R * \text{pow}(Q_c * F, 2) + Q_c * (1 - F) * (Q_c * F + Q_p / P_B) * R) / \\ &\quad ((Q_c + L) * (Q_c * F + Q_p / P_B) * (Q_c + F * k_G) - Q_c * (1 - F) * \text{pow}(Q_c * F, 2) - (Q_c * F + Q_p / P_B) * \text{pow}(Q_c * (1 - F), 2)) \end{aligned}$$

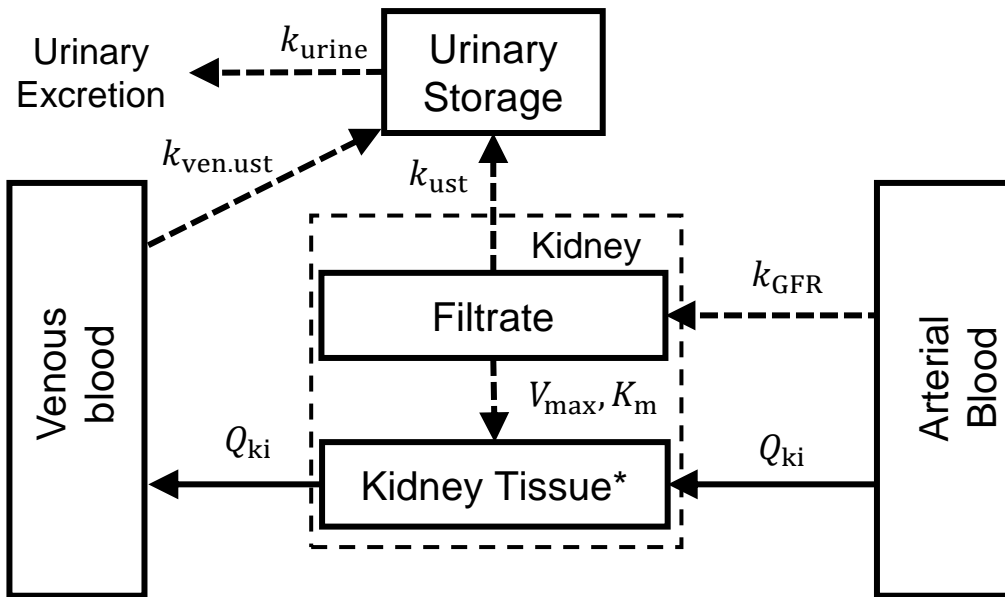
If $F=1$, $k_G=0$, $L=0$:

$$\begin{aligned} C_{ven} &= ((Q_c + 1 * 0) * (Q_c * 1 + Q_p / P_B) * R + Q_c * 1 * Q_c * (1 - 1) * Q_{in}) / \\ &\quad ((Q_c + 0) * (Q_c * 1 + Q_p / P_B) * (Q_c + 1 * 0) - Q_c * (1 - 1) * \text{pow}(Q_c * 1, 2) - (Q_c * 1 + Q_p / P_B) * \text{pow}(Q_c * (1 - 1), 2)) \\ &= ((Q_c) * (Q_c + Q_p / P_B) * R) / ((Q_c) * (Q_c + Q_p / P_B) * (Q_c)) = R / Q_c \\ C_{pulv} &= ((Q_c + 1 * 0) * (Q_c + 0) * Q_{in} + (Q_c + 1 * 0) * Q_c * 1 * R - Q_{in} * \text{pow}(Q_c * (1 - 1), 2)) / \\ &\quad ((Q_c + 0) * (Q_c * 1 + Q_p / P_B) * (Q_c + 1 * 0) - Q_c * (1 - 1) * \text{pow}(Q_c * 1, 2) - (Q_c * 1 + Q_p / P_B) * \text{pow}(Q_c * (1 - 1), 2)) \\ &= ((Q_c) * (Q_c) * Q_{in} + (Q_c) * Q_c * R) / ((Q_c) * (Q_c + Q_p / P_B) * (Q_c)) = (Q_{in} + R) / ((Q_c + Q_p / P_B)) \\ &= (Q_{in} + Q_c * C_{ven}) / ((Q_c + Q_p / P_B)) \\ C_{art} &= (Q_c * 1 * (Q_c + 0) * Q_{in} + R * \text{pow}(Q_c * 1, 2) + Q_c * (1 - 1) * (Q_c * 1 + Q_p / P_B) * R) / \\ &\quad ((Q_c + 0) * (Q_c * 1 + Q_p / P_B) * (Q_c + 1 * 0) - Q_c * (1 - 1) * \text{pow}(Q_c * 1, 2) - (Q_c * 1 + Q_p / P_B) * \text{pow}(Q_c * (1 - 1), 2)) \\ &= (Q_c * (Q_c) * Q_{in} + R * \text{pow}(Q_c, 2)) / ((Q_c) * (Q_c + Q_p / P_B) * (Q_c)) = (Q_{in} + R) / (Q_c + Q_p / P_B) = C_{pulv} \end{aligned}$$

Template Model



Kidney Subfigure



Liver and GI Subfigure

